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SHORTER COMMUNICATIONS

EFFECT OF WALL HEAT CONDUCTION ON CONVECTION IN A CIRCULAR TUBE WITH ARBITRARY CIRCUMFERENTIAL HEAT INPUT

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IN AN earlier paper [1] the author presented an analysis of the fully developed convective processes for laminar or turbulent flow in a circular tube with arbitrary circumferential wall heat flux. In practical situations one may know the circumferential heat input to the tube, but the conduction processes within the wall tend to smooth out any variations, making the circumferential heat flux variation from the tube to the fluid less drastic. However, the results of the earlier analysis may be applied immediately in consideration of the wall conduction effects, and this means is developed below.

The differential equation describing one-dimensional heat conduction within a uniform thickness tube is*

$$\frac{k_w \delta}{r_o^2} \frac{\partial^2 t_w}{\partial \theta^2} - q_w''(\theta) + q_s''(\theta) = 0. \qquad (1)$$

The nomenclature of [1] has been adopted, with the addition of k_w for the wall conductivity, δ for wall thickness, and q_s'' for the known heat flux input to the tube. q_w'' represents the convection per unit area from the tube to the fluid, which was considered known in [1]. We assume that q_s'' is known in the form of a Fourier expansion,

$$q_s''(\theta) = q_{so}'' + \sum_{n=1}^{\infty} (A_n \sin n\theta + B_n \cos n\theta). \qquad (2)$$

In [1] the wall heat flux is expressed in the form

$$q_w''(\theta) = q_o'' + \sum_{n=1}^{\infty} (a_n \sin n\theta + b_n \cos n\theta). \qquad (3)$$

It is an easy matter to relate the coefficients a_n and b_n to the known coefficients A_n and B_n. From equation (17) of [1] we can express the temperature distribution in terms of the a_n's, b_n's, and known functions S_n. Assuming that this temperature distribution is twice-differentiable, we differentiate and combine with (1) above, obtaining

$$-\frac{k_w \delta}{k r_o} \left[\sum_{n=1}^{\infty} S_n n^2 (a_n \sin n\theta + b_n \cos n\theta) \right]$$
$$- \left[q_o'' + \sum_{n=1}^{\infty} (a_n \sin n\theta + b_n \cos n\theta) \right]$$
$$+ \left[q_{so}'' + \sum_{n=1}^{\infty} (A_n \sin n\theta + B_n \cos n\theta) \right] = 0. \qquad (4)$$

Integrating around the circumference, it follows that

$$q_o'' = q_{so}''. \qquad (5a)$$

Multiplying by $\sin m\theta$, and integrating around the circumference, only the terms going with $n = m$ are retained. We then find

$$a_n = A_n \Big/ \left[1 + S_n n^2 \left(\frac{k_w \delta}{k r_o} \right) \right]. \qquad (5b)$$

Similarly, multiplying by $\cos m\theta$ and integrating,

$$b_n = B_n \Big/ \left[1 + S_n n^2 \left(\frac{k_w \delta}{k r_o} \right) \right]. \qquad (5c)$$

Knowing the coefficients a_n and b_n, we can return to equation (17) of [1] to find the temperature distribution.

$$t_w(\theta, x) - t_m(x) = \frac{r_o}{k} \left\{ S_o q_{so}'' + \sum_{n=1}^{\infty} \frac{S_n}{[1 + S_n n^2 (k_w \delta / k r_o)]} (A_n \sin n\theta + B_n \cos n\theta) \right\}. \qquad (6)$$

In particular, for the example of [1],

$$\frac{t_w - t_m}{q_{so}'' r_o / k} = 0{\cdot}0112 \left\{ 1 + \frac{0{\cdot}80 \cos \theta}{[1 + 0{\cdot}018 (k_w \delta / k r_o)]} \right\}. \qquad (7)$$

* Axial conduction vanishes in thermally fully developed flow. Radial gradients are neglected in the one-dimensional analysis.

REFERENCE

1. W. C. REYNOLDS, Turbulent heat transfer in a circular tube with variable circumferential heat flux, *Int. J. Heat Mass Transfer* **6**, 445 (1963).